

Finding the Lovasz Number

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November 25, 2024

The Lovasz Number of a graph \mathbf{G} , denoted $\vartheta(\mathbf{G})$, is the upper bound on the Shannon capacity of the graph ([1]). For an adjacency matrix $\mathbf{B} = [B_{ij}]$ the problem of finding the Lovasz number is given by the following primal SQLP problem

$$\begin{aligned} & \underset{\mathbf{X}}{\text{minimize}} && \text{tr}(\mathbf{C}\mathbf{X}) \\ & \text{subject to} && \text{tr}(\mathbf{X}) = 1 \\ & && X_{ij} = 0 \text{ if } B_{ij} = 1 \\ & && \mathbf{X} \in \mathcal{S}^n \end{aligned}$$

The function `lovasz` takes as input an adjacency matrix `B`, and returns the the optimal Lovasz number using `sqlp`.

```
R> out <- lovasz(B)
```

Numerical Example

To compute the Lovasz number using `sqlp`, we need only the (weighted) adjacency matrix representing a graph object.

```
R> data(Glovasz)
```

```
      V1 V2 V3 V4 V5 V6 V7 V8 V9 V10
[1,]  0  0  0  1  0  0  1  1  0  0
[2,]  0  0  0  1  0  0  1  0  1  1
[3,]  0  0  0  0  0  0  0  0  1  0
[4,]  1  1  0  0  0  0  0  0  1  0
[5,]  0  0  0  0  0  0  0  1  1  1
[6,]  0  0  0  0  0  0  0  0  0  1
[7,]  1  1  0  0  1  0  0  1  1  1
[8,]  1  0  1  1  1  0  1  0  0  0
[9,]  0  1  0  0  1  1  1  0  0  1
[10,] 0  1  0  1  1  0  1  0  1  0
```

The Lovasz number for the associated graph is the value of the primal objective function. Again, since the objective function was negated to make the primal problem a minimization, we negate the value of the objective function.

```
R> out <- lovasz(Glovasz)
```

```
R> -out$pobj
```

```
[1] 5
```

References

- [1] László Lovász. On the shannon capacity of a graph. *IEEE Transactions on Information theory*, 25(1):1–7, 1979.